Экзаменационные задачи по электродинамике сплошных сред (6 семестр)

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N 10.1. стр 1

а) $h = \sqrt{\frac{\omega^2}{c^2}\varepsilon\mu - \left(\frac{\pi}{a}\right)^2}$, $\varepsilon, \mu = 1$

$\omega_{cr} = \frac{\pi c}{a} = \frac{3{,}14 \cdot 3 \cdot 10^{10}\ \frac{см}{с}}{2\ см} = 4{,}71 \cdot 10^{10}\ \frac{1}{с}$, $\lambda_{cr} = \frac{2\pi c}{\omega_{cr}} = 2a = 4 см.$

$\lambda_g = \frac{2\pi}{h} = \frac{2\pi c}{\omega}\left(1 - \frac{\omega_{cr}^2}{\omega^2}\right)^{-\frac{1}{2}} = 4{,}53\ см.$

$v = \frac{\omega}{h} = c\left(1 - \frac{\omega_{cr}^2}{\omega^2}\right)^{-\frac{1}{2}} = 4{,}53 \cdot 10^{10}\ \frac{см}{с}$

$\frac{\omega^2}{c^2} = h^2 + \left(\frac{\pi}{a}\right)^2$, $\frac{1}{c^2} 2\omega d\omega = 2h\,dh$, $v_g = \frac{d\omega}{dh} = \frac{c^2}{\frac{\omega}{h}} = \frac{c^2}{v} = 1{,}98 \cdot 10^{10}\ \frac{см}{с}$

б) $\left.\begin{array}{l} H_z = \frac{\varkappa^2}{ik_0\varepsilon\mu}\psi^m \\ \overline{H}_\perp = -\frac{h}{k_0\varepsilon\mu}\nabla_\perp\psi^m \\ \overline{E}_\perp = -\frac{1}{\varepsilon}[\nabla_\perp\psi^m, \overline{z}^0] \\ E_z = 0 \end{array}\right\} e^{i(\omega t - hz)}$

$\frac{\partial^2}{\partial x^2}\psi^m + \varkappa^2\psi^m = 0$,

$\left.\frac{\partial\psi^m}{\partial n}\right|_e = 0.$

$\psi^m = c_1 \cos\frac{\pi x}{a}$

$\varepsilon, \mu = 1$, $\overline{E}_\perp = c_1\frac{\pi}{a}\sin\frac{\pi x}{a}[\overline{x}^0, \overline{z}^0]e^{i(\omega t - hz)} =$

$= \overline{y}_0\left(-c_1\frac{\pi}{a}\right)\sin\frac{\pi x}{a}e^{i(\omega t - hz)} = \overline{y}_0 E_{max}\sin\frac{\pi x}{a}e^{i(\omega t - hz)}$,

$\overline{H}_\perp = \overline{x}_0\frac{h}{k_0}c_1\frac{\pi}{a}\sin\frac{\pi x}{a}e^{i(\omega t - hz)} = -\overline{x}_0\frac{h}{k_0}E_{max}\sin\frac{\pi x}{a}e^{i(\omega t - hz)} =$

$= \overline{x}_0 H_{max_\perp}\sin\frac{\pi x}{a}e^{i(\omega t - hz)}$, $|H_{max_\perp}| = E_{max}\left(1 - \frac{\omega_{cr}^2}{\omega^2}\right)^{\frac{1}{2}}$

$\operatorname{div}\overline{H} = 0 = \operatorname{div}(\overline{H}_\perp + \overline{H}_z)$, $\frac{\partial H_z}{\partial z} = -\frac{\partial H_\perp}{\partial x}$

$H_z = i H_{max_z}\cos\frac{\pi x}{a}e^{i(\omega t - hz)}$

$H_\perp = H_{max_\perp}\sin\frac{\pi x}{a}e^{i(\omega t - hz)}$

$h H_{max_z} = -\frac{\pi}{a}H_{max_\perp} = \frac{\pi}{a}\frac{h}{k_0}E_{max}.$

$H_{max_z} = \frac{\frac{\pi c}{a}}{k_0 c}E_{max} = E_{max}\frac{\omega_{cr}}{\omega}$

N10.2.

Если два импульса были одновременно отправлены друг другу навстречу и их центры совпали посередине волновода, значит у них одинаковые v_g.

$h^2 = \frac{\omega^2}{c^2} - \varkappa^2$, $\frac{2\omega d\omega}{c^2} = 2h dh$, $v_g = \frac{d\omega}{dh} = \frac{c^2}{v}$

$v_{g_1} = \frac{c^2}{v_1} = \frac{c^2}{\omega_1} h_{10}$, $v_{g_2} = \frac{c^2}{v_2} = \frac{c^2}{\omega_2} h_{m0}$,

$h_{10} = \sqrt{\frac{\omega_1^2}{c^2} - \left(\frac{\pi}{a}\right)^2}$, $h_{m0} = \sqrt{\frac{\omega_2^2}{c^2} - \left(\frac{m\pi}{a}\right)^2}$,

$\frac{h_{10}}{\omega_1} = \frac{h_{m0}}{\omega_2}$, $\sqrt{\frac{1}{c^2} - \left(\frac{\pi}{a\omega_1}\right)^2} = \sqrt{\frac{1}{c^2} - \left(\frac{m\pi}{a\omega_2}\right)^2}$

$\Rightarrow \frac{1}{\omega_1} = \frac{m}{\omega_2}$, $\frac{\lambda_1}{2\pi c} = \frac{\lambda_2 m}{2\pi c}$, $\frac{\lambda_1}{\lambda_2} = m$.

В свободном пространстве: $\lambda^{(0)} = \frac{2\pi}{k}$, $\varepsilon, \mu = 1$,

$\lambda_1^{(0)} = \frac{2\pi c}{\omega_1}$, $\lambda_2^{(0)} = \frac{2\pi c}{\omega_2}$ $\Rightarrow$ $\frac{\lambda_1^{(0)}}{\lambda_2^{(0)}} = m$

N 10.3. $h = \sqrt{\frac{\omega^2}{c^2} - æ^2}$

$$\lambda_g = \frac{2\pi}{h} = \frac{2\pi c}{\omega}\left(1 - \frac{\omega_{cr}^2}{\omega^2}\right)^{-\frac{1}{2}},$$

$$\omega_{cr} = æ c = \frac{2\pi c}{\lambda_{cr}}, \quad \lambda_{cr} = \frac{2\pi}{æ}.$$

$$\lambda_g = 2\lambda_{cr}, \quad \sqrt{1 - \frac{\omega_{cr}^2}{\omega^2}}\,\frac{4\pi}{æ} = \frac{2\pi c}{\omega},$$

$$\left(1 - \frac{\omega_{cr}^2}{\omega^2}\right) = \frac{æ^2 c^2}{4\omega^2}, \quad \omega^2 - \omega_{cr}^2 = \frac{æ^2 c^2}{4},$$

$$\frac{\omega^2}{\omega_{cr}^2} = \frac{1}{4} + 1 = \frac{5}{4}, \quad \frac{\omega}{\omega_{cr}} = \frac{\sqrt{5}}{2}.$$

N 10.5.

а) TE_{10}, $a > b$., $h^2 = \frac{\omega^2}{c^2} - \left(\frac{\pi}{a}\right)^2$, $\frac{2\omega d\omega}{c^2} = 2h\,dh$

$$v_g = \frac{d\omega}{dh} = \frac{c^2}{v} = \frac{c^2}{\omega} h_{10} = c\left(1 - \frac{\omega_{cr}^2}{\omega^2}\right)^{\frac{1}{2}}.$$

$$t = \frac{L}{v_g} = \frac{L}{c}\left(1 - \frac{\omega_{cr}^2}{\omega^2}\right)^{-\frac{1}{2}}$$

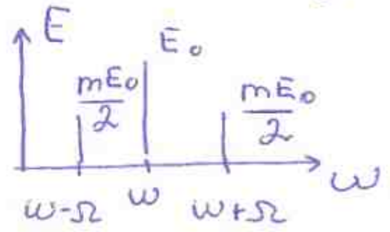
б) TEM

$$v = \frac{\omega}{k} = \frac{c}{\sqrt{\varepsilon\mu}}, \quad \varepsilon, \mu = 1, \quad v = c., \quad t = \frac{L}{c}.$$

№ 10.8

TE_{10}, $E = E_0(1 + m\cos\Omega t)\,e^{i\omega t} =$

$= E_0 e^{i\omega t} + \frac{mE_0}{2} e^{i(\Omega+\omega)t} + \frac{mE_0}{2} e^{i(\omega-\Omega)t}$ – на входе в волновод



В волноводе: $E_{\omega+\Omega} = \frac{mE_0}{2} e^{i[(\omega+\Omega)t - h_{\omega+\Omega} z]}$,

$E_{\omega-\Omega} = \frac{mE_0}{2} e^{i[(\omega-\Omega)t - h_{\omega-\Omega} z]}$,

$E_\omega = E_0 e^{i(\omega t - h_\omega z)}$.

По условию $\frac{\pi c}{\omega} > a > \frac{\pi c}{\omega+\Omega}$, $\frac{\pi c}{a} = \omega_{cr}$,

$\omega,\ \omega - \Omega < \omega_{cr}$. Значит, $E_{\omega-\Omega}$ и E_ω являются нераспространяющимися волнами.

E_ω: $h_\omega = \pm i\sqrt{\frac{\pi^2}{a^2} - \frac{\omega^2}{c^2}}$

$E_\omega = E_0 e^{i\omega t \pm |h_\omega| z}$

Амплитуда волны экспоненциально возрастает или затухает.

$E_{\omega-\Omega}$: $h_{\omega-\Omega} = \pm i\sqrt{\frac{\pi^2}{a^2} - \frac{(\omega-\Omega)^2}{c^2}}$

$E_{\omega-\Omega} = \frac{mE_0}{2} e^{i(\omega-\Omega)t \pm |h_{\omega-\Omega}| z}$

Амплитуда экспоненциально возрастает или затухает.

$E_{\omega+\Omega}$: $h_{\omega+\Omega} = \sqrt{\frac{(\omega+\Omega)^2}{c^2} - \frac{\pi^2}{a^2}}$

$E_{\omega+\Omega} = \frac{mE_0}{2} e^{i\left[(\omega+\Omega)t - \sqrt{\frac{(\omega+\Omega)^2}{c^2} - \frac{\pi^2}{a^2}}\, z\right]}$

Амплитуда не изменяется.

Спектр частот в волноводе.

N 10.4. (a)

стр 5

$a = 10$ см., $b = 7$ см., $f = \frac{\omega}{2\pi}$, $\bar{E}_\perp = -\frac{1}{\varepsilon}[\nabla_\perp \psi^m, \bar{z}^0] e^{i(\omega t - hz)}$,

$\frac{\partial^2}{\partial x^2}\psi^m + æ^2\psi^m = 0$, $\frac{\partial \psi^m}{\partial n}\Big|_e = 0$.

a) TE_{10} и TE_{30} при $f = 1700$ Мгц.

TE_{10}: $E_y^{10} = -C_{10}\frac{\pi}{a}\sin\frac{\pi x}{a} e^{i(\omega t - h_{10} z)}$.

$h_{10} = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{\pi}{a}\right)^2}$, $\omega_{cr} = \frac{\pi c}{a} = 9424$ Мгц

$f_{cr} = 1500$ Мгц.

TE_{30}: $E_y^{30} = -C_{30}\frac{3\pi}{a}\sin\frac{3\pi x}{a} e^{i(\omega t - h_{30} z)}$.

$h_{30} = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{3\pi}{a}\right)^2}$, $\omega_{cr} = \frac{3\pi c}{a}$, $f_{cr} = 4500$ Мгц.

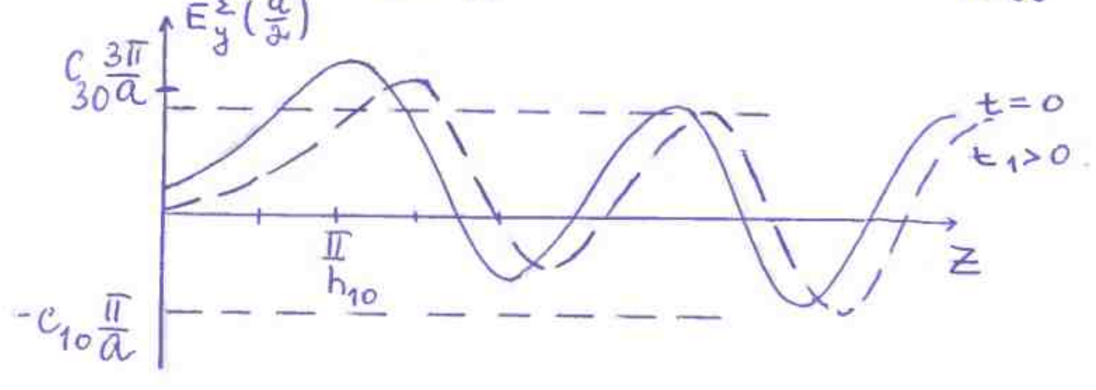
$\Rightarrow h_{30} = \pm i\sqrt{\left(\frac{3\pi}{a}\right)^2 - \frac{\omega^2}{c^2}} \Rightarrow E_y^{30} = -C_{30}\frac{3\pi}{a}\sin\frac{3\pi x}{a} e^{-|h_{30}|z} e^{i\omega t}$

Выбрали h так, чтобы амплитуда волны экспоненциально затухала.

$E_y^{\Sigma} = E_y^{10} + E_y^{30} = -\left(C_{10}\frac{\pi}{a}\sin\frac{\pi x}{a} e^{-ih_{10}z} + C_{30}\frac{3\pi}{a}\sin\frac{3\pi x}{a} e^{-|h_{30}|z}\right) \cdot e^{i\omega t}$

$E_y^{\Sigma}\left(\frac{a}{2}\right) = -C_{10}\frac{\pi}{a} e^{i(\omega t - h_{10} z)} + C_{30}\frac{3\pi}{a} e^{-|h_{30}|z} e^{i\omega t}$

$\mathrm{Re}\, E_y^{\Sigma}\left(\frac{a}{2}\right) = C_{30}\frac{3\pi}{a} e^{-|h_{30}|z}\cos\omega t - C_{10}\frac{\pi}{a}\cos(\omega t - h_{10} z)$



№ 10.4. (б)

$a = 10$ см, $b = 7$ см., $f = \frac{\omega}{2\pi}$, $\bar{E}_\perp = -\frac{1}{\varepsilon}[\nabla_\perp \psi^m, \bar{z}^0] e^{i(\omega t - hz)}$,

$\frac{\partial^2}{\partial x^2}\psi^m + æ^2 \psi^m = 0$, $\left.\frac{\partial \psi^m}{\partial n}\right|_e = 0$.

б) TE_{10} и TE_{20} при $f = 1700$ Мгц.

TE_{10}: $E_y^{10} = -C_{10}\frac{\pi}{a}\sin\frac{\pi x}{a} e^{i(\omega t - h_{10} z)}$

$h_{10} = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{\pi}{a}\right)^2}$, $\omega_{cr} = \frac{\pi c}{a}$, $f_{cr} = \frac{\omega_{cr}}{2\pi} = 1500$ Мгц.

TE_{20}: $E_y^{20} = -C_{20}\frac{2\pi}{a}\sin\frac{2\pi x}{a} e^{i(\omega t - h_{20} z)}$

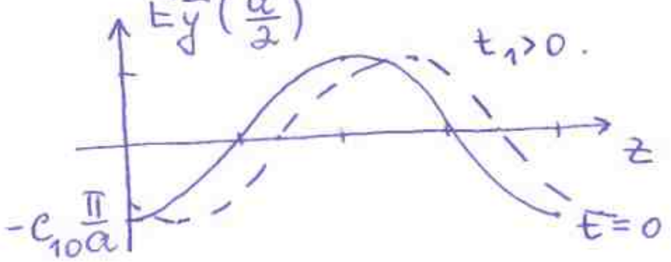
$h_{20} = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{2\pi}{a}\right)^2}$, $\omega_{cr} = \frac{2\pi c}{a}$, $f_{cr} = \frac{c}{a} = 3000$ Мгц.

$\Rightarrow h_{20} = \pm i\sqrt{\left(\frac{2\pi}{a}\right)^2 - \frac{\omega^2}{c^2}} \Rightarrow E_y^{20} = -C_{20}\frac{2\pi}{a}\sin\frac{2\pi x}{a} e^{-|h_{20}|z} e^{i\omega t}$

Выбрали h так, чтобы амплитуда волны экспоненциально затухала.

$E_y^{\Sigma} = E_y^{10} + E_y^{20}$.

$\mathrm{Re}\, E_y^{\Sigma}\left(\frac{a}{2}\right) = -C_{10}\frac{\pi}{a}\cos(\omega t - h_{10} z)$



В сечении $x \neq \frac{a}{2}$ график будет такой же, как в 10.4 (а)

N 10.4 (в)

$a = 10$ см, $b = 7$ см, $f = \frac{\omega}{2\pi}$, $\bar{E}_\perp = -\frac{1}{\varepsilon}[\nabla_\perp \psi^m, \bar{z}^0] e^{i(\omega t - hz)}$,

$\frac{\partial^2}{\partial x^2}\psi^m + \varkappa^2 \psi^m = 0$, $\left.\frac{\partial \psi^m}{\partial n}\right|_e = 0$.

в) TE_{10} и TE_{30} с одинаковыми амплитудами поля на оси при $f = 10^5$ Мгц.

TE_{10}: $E_y^{10} = -C_{10}\frac{\pi}{a}\sin\frac{\pi x}{a} e^{i(\omega t - hz)}$

$h = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{\pi}{a}\right)^2}$, $\omega_{cr} = \frac{\pi c}{a}$, $f_{cr} = \frac{\omega_{cr}}{2\pi} = 1500$ Мгц.

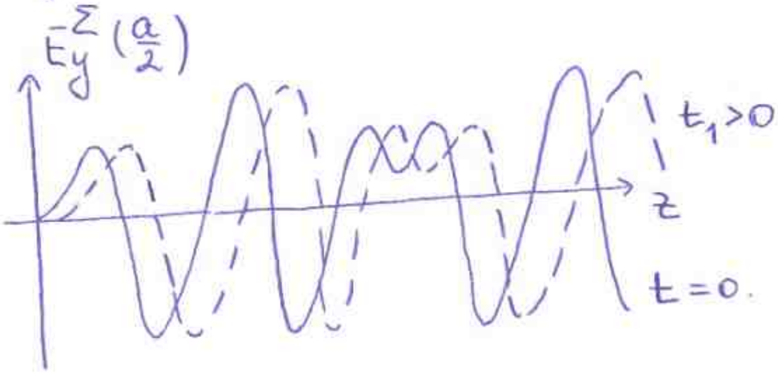
TE_{30}: $E_y^{30} = -C_{30}\frac{3\pi}{a}\sin\frac{3\pi x}{a} e^{i(\omega t - hz)}$

$h = \sqrt{\frac{\omega^2}{c^2} - \left(\frac{3\pi}{a}\right)^2}$, $\omega_{cr} = \frac{3\pi c}{a}$, $f_{cr} = 4500$ Мгц.

Обе волны распространяются без затухания.

$E_y^\Sigma = E_y^{10} + E_y^{30}$, $C_{10}\frac{\pi}{a} = C_{30}\frac{3\pi}{a} = C$ (по условию)

$\mathrm{Re}\, E_y^\Sigma\left(\frac{a}{2}\right) = C\left(\cos(\omega t - h_{30} z) - \cos(\omega t - h_{10} z)\right)$



$N\,10.16.$ стр 8

TE_{10}, $a>b$, $\varkappa = \frac{\pi}{a}$

$$\left.\begin{aligned} H_z &= \frac{\varkappa^2}{ik_0\varepsilon\mu}\psi^m \\ \bar{H}_\perp &= -\frac{h}{k_0\varepsilon\mu}\nabla_\perp\psi^m \\ \bar{E}_\perp &= -\frac{1}{\varepsilon}[\nabla_\perp\psi^m, \bar{z}^0] \\ E_z &= 0 \end{aligned}\right\} e^{i(\omega t - hz)} \quad \left| \quad \begin{aligned} &\frac{\partial^2}{\partial x^2}\psi^m + \varkappa^2\psi^m = 0, \\ &\left.\frac{\partial\psi^m}{\partial n}\right|_e = 0. \\ &\psi^m = C_1\cos\frac{\pi x}{a}. \end{aligned}\right.$$

$$\varepsilon,\mu = 1,\quad \bar{E}_\perp = C_1\frac{\pi}{a}\sin\frac{\pi x}{a}[\bar{x}^0, \bar{z}^0]e^{i(\omega t - hz)} = \bar{y}_0\left(-C_1\frac{\pi}{a}\right)\sin\frac{\pi x}{a}e^{i(\omega t - hz)} =$$

$$= \bar{y}_0 E_0\sin\frac{\pi x}{a}e^{i(\omega t - hz)},\quad \bar{H}_\perp = \bar{x}_0\frac{h}{k_0}C_1\frac{\pi}{a}\sin\frac{\pi x}{a}e^{i(\omega t - hz)} =$$

$$= -\bar{x}_0\frac{h}{k_0}E_0\sin\frac{\pi x}{a}e^{i(\omega t - hz)},\quad H_z = \frac{\left(\frac{\pi}{a}\right)^2}{ik_0}C_1\cos\frac{\pi x}{a}e^{i(\omega t - hz)} =$$

$$= -\frac{\pi E_0}{ik_0 a}\cos\frac{\pi x}{a}e^{i(\omega t - hz)}$$

$$\zeta_{\perp b} = \frac{k_0}{h} \quad (\text{если } \varepsilon,\mu = 1)$$

$$\zeta_{\text{пов}} = \sqrt{\frac{\mu_\kappa}{\varepsilon_\kappa}} = \sqrt{\frac{i\omega}{4\pi\sigma}} = (i+1)\sqrt{\frac{\omega}{8\pi\sigma}}$$

$$|h''| = \frac{\operatorname{Re}\zeta_{\text{пов}}\oint_L |H_\tau|^2 dl}{2\operatorname{Re}\zeta_{\perp b}\iint |H_\perp|^2 dS}$$

$$\bar{H}_\tau = \bar{H}_\perp + \bar{H}_z,\quad |H_\tau|^2 = H_\perp H_\perp^* + H_z H_z^* = \frac{h^2}{k_0^2}E_0^2\sin^2\frac{\pi x}{a} + \frac{\pi^2 E_0^2}{k_0^2 a^2}\cos^2\frac{\pi x}{a}$$

$$\oint_L |H_\tau|^2 dl = \int_0^a \frac{E_0^2}{k_0^2}\left(h^2\sin^2\frac{\pi x}{a} + \frac{\pi^2}{a^2}\cos^2\frac{\pi x}{a}\right)dx = \frac{E_0^2}{k_0^2}\cdot\frac{a}{2}\left(h^2 + \frac{\pi^2}{a^2}\right) = \frac{E_0^2 a}{2}.$$

$$\iint |H_\perp|^2 dS = \int_0^a\int_0^b \left(\frac{h}{k_0}E_0\sin\frac{\pi x}{a}\right)^2 dx\,dy = \frac{h^2E_0^2 ba}{2k_0^2}$$

$$|h''| = \frac{\sqrt{\frac{\omega}{8\pi\sigma}}}{2\frac{k_0}{h}}\,\frac{\frac{E_0^2 a}{2}}{\frac{h^2E_0^2 ba}{2k_0^2}} = \frac{k_0}{2hb}\sqrt{\frac{\omega}{8\pi\sigma}}$$

N 10.18 (б) стр 9

$\varepsilon,\mu=1$, $H_\varphi = \frac{E_0}{r} e^{i(\omega t - kz)}$, $E_r = \frac{E_0}{r} e^{i(\omega t - kz)}$

$$\zeta_{пов} = \sqrt{\frac{i\omega}{4\pi\sigma}} = (i+1)\sqrt{\frac{\omega}{8\pi\sigma}}$$

$$\zeta_{\perp в} = \sqrt{\frac{\mu}{\varepsilon}} = 1$$

$$|h''| = \frac{\mathrm{Re}\,\zeta_{пов} \oint\limits_L |H_\tau|^2 dl}{2\mathrm{Re}\,\zeta_{\perp в} \iint |H_\perp|^2 ds}$$

$$\oint\limits_L |H_\tau|^2 dl = 2\pi\left(\frac{1}{a} + \frac{1}{b}\right)E_0^2$$

$$\iint |H_\perp|^2 ds = 2\pi \ln\frac{b}{a} E_0^2$$

$$|h''| = \sqrt{\frac{\omega}{8\pi\sigma}} \frac{(a+b)}{2ab}\left(\ln\frac{b}{a}\right)^{-1}$$

N 10.19

TE_{10}, $a>b$, $\varepsilon,\mu=1$

$$\left.\begin{aligned} H_z &= \frac{\varkappa^2}{ik_0\varepsilon\mu}\psi^m \\ \bar{H}_\perp &= -\frac{h}{k_0\varepsilon\mu}\nabla_\perp\psi^m \\ \bar{E}_\perp &= -\frac{1}{\varepsilon}[\nabla_\perp\psi^m, \bar{z}^0] \\ E_z &= 0 \end{aligned}\right\} e^{i(\omega t - hz)}$$

$$\frac{\partial^2}{\partial x^2}\psi^m + \varkappa^2\psi^m = 0,\quad \left.\frac{\partial\psi^m}{\partial n}\right|_l = 0.\quad \psi^m = C_1\cos\frac{\pi x}{a}$$

$$\bar{E}_\perp = \bar{y}_0\left(-C_1\frac{\pi}{a}\right)\sin\frac{\pi x}{a} e^{i(\omega t - kz)} = \bar{y}_0 E_{max}\sin\frac{\pi x}{a} e^{i(\omega t - kz)},$$

$$\bar{H}_\perp = \bar{x}_0\frac{h}{k_0}C_1\frac{\pi}{a}\sin\frac{\pi x}{a}e^{i(\omega t - kz)} = -\bar{x}_0\frac{1}{\zeta_{\perp в}}E_{max}\sin\frac{\pi x}{a}e^{i(\omega t - kz)}.$$

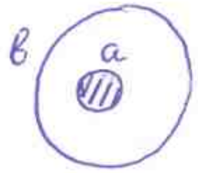
$$P = \frac{c}{8\pi}\mathrm{Re}\,\zeta_{\perp в}\iint|H_\perp|^2 ds = \frac{c}{8\pi}\frac{E_{max}^2}{\zeta_{\perp в}}\frac{ab}{2},$$

$$E_{max} = \left(\frac{16\pi k_0 P}{c h a b}\right)^{\frac{1}{2}},\quad \bar{H} = H_z\bar{z}^0 + \bar{H}_\perp = -\bar{z}^0\frac{\frac{\pi}{a}E_{max}}{ik_0}\cos\frac{\pi x}{a}\cdot e^{i(\omega t - hz)} - \bar{x}_0\frac{1}{\zeta_{\perp в}}E_{max}\sin\frac{\pi x}{a}e^{i(\omega t - hz)}.$$

$$H_{max} = \sqrt{H_\perp^2 - H_z^2} = \frac{E_{max}}{k_0}\max\{h, \varkappa\}$$

N 10.22.

а)



$$\oint\!\!\!\oint_S \bar{D}d\bar{S} = 4\pi q, \quad D_r 2\pi r l = 4\pi q_{лин} l,$$

$\varepsilon, \mu = 1.$

$$D_r = \frac{2q_{лин}}{r} = E_r, \quad V = \int_a^b E_r dr = 2q_{лин} \ln\left(\frac{b}{a}\right),$$

$$C_{пог} = \frac{q_{лин}}{V} = \frac{1}{2\ln\left(\frac{b}{a}\right)}.$$

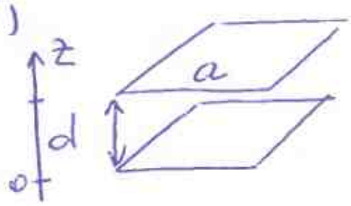
$$\oint_L H dl = \frac{4\pi}{c} I, \quad H_\varphi 2\pi r = \frac{4\pi}{c} I, \quad H_\varphi = \frac{2I}{cr} = B_\varphi.$$

$$\Phi = \iint B_\varphi dS = l\int_a^b B_\varphi dr = l\frac{2I}{c}\ln\left(\frac{b}{a}\right), \quad \Phi = \frac{LI}{c},$$

$$L = 2l\ln\left(\frac{b}{a}\right), \quad L_{пог} = 2\ln\left(\frac{b}{a}\right)$$

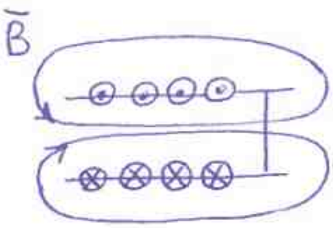
$$Z_b = \frac{1}{c}\sqrt{\frac{L_{пог}}{C_{пог}}} = \frac{2}{c}\ln\left(\frac{b}{a}\right)$$

б)



$$\oint\!\!\!\oint_S \bar{D}d\bar{S} = 4\pi q, \quad D_n a l = 4\pi \sigma a l,$$

$$D_n = 4\pi\sigma = E_n, \quad V = \int_0^d E_n dz = 4\pi\sigma d$$



$$C_{пог} = \frac{q_{лин}}{V} = \frac{\sigma a}{4\pi\sigma d} = \frac{a}{4\pi d}.$$

$$\oint_L H dl = \frac{4\pi I}{c}, \quad Ha = \frac{4\pi I}{c}, \quad H = \frac{4\pi I}{ca}.$$

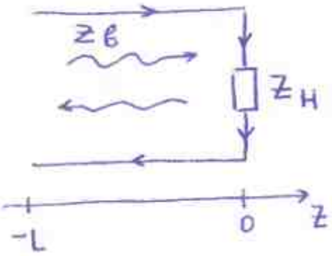
$$\Phi = \iint B_n dS = l\int_0^d \frac{4\pi I}{ca} dz = \frac{4\pi I}{ca} ld,$$

$$\Phi = \frac{LI}{c}, \quad L = \frac{4\pi l d}{a}, \quad L_{пог} = \frac{4\pi d}{a}.$$

$$Z_b = \frac{1}{c}\sqrt{\frac{L_{пог}}{C_{пог}}} = \frac{4\pi d}{ac}$$

стр 11

N 10.23



$$\begin{cases} V(z) = V_i\, e^{-ikz} + V_r\, e^{ikz} \\ I(z) = I_i e^{-ikz} + I_r e^{ikz}. \end{cases}, \quad \Big| \quad \frac{V_i}{I_i} = z_в, \ \frac{V_r}{I_r} = -Z_в$$

$$I(z) = \frac{V_i}{z_в} e^{-ikz} - \frac{V_r}{z_в} e^{ikz}$$

$$Z(z) = \frac{V(z)}{I(z)} = \frac{V_i e^{-ikz} + V_r e^{ikz}}{\frac{V_i}{z_в} e^{-ikz} - \frac{V_r}{z_в} e^{ikz}} =$$

$$= z_в \frac{e^{-ikz} + \frac{V_r}{V_i} e^{ikz}}{e^{-ikz} - \frac{V_r}{V_i} e^{ikz}} = z_в \frac{e^{-ikz} + \Gamma e^{ikz}}{e^{-ikz} - \Gamma e^{ikz}}$$

при $z = 0$: $\quad Z(0) = Z_н = z_в \dfrac{1+\Gamma}{1-\Gamma} \Rightarrow \boxed{\Gamma = \dfrac{z_н - z_в}{z_н + z_в}}$

$$Z(z) = z_в \frac{e^{-ikz} + \Gamma e^{ikz}}{e^{-ikz} - \Gamma e^{ikz}} = z_в \frac{\cos kz - i\sin kz + \Gamma(\cos kz + i\sin kz)}{\cos kz - i\sin kz - \Gamma(\cos kz + i\sin kz)}$$

$$= Z_в \frac{\cos kz(1+\Gamma) - i\sin kz(1-\Gamma)}{\cos kz(1-\Gamma) - i\sin kz(1+\Gamma)} = z_в \frac{z_н - i z_в \operatorname{tg} kz}{z_в - i z_н \operatorname{tg} kz}$$

$$\boxed{Z(-L) = z_в \frac{z_н + i z_в \operatorname{tg} kL}{z_в + i z_н \operatorname{tg} kL}}$$

а) $z_н = \dfrac{1}{i\omega C}$, $|\Gamma| = \dfrac{|1 - i\omega C z_в|}{|1 + i\omega C z_в|} = 1$, $Z(-L) = Z_в \dfrac{\frac{1}{i\omega C} + i z_в \operatorname{tg} kL}{z_в + \frac{1}{\omega C}\operatorname{tg} kL} =$

$$= i z_в \frac{z_в \omega C \operatorname{tg} kL - 1}{z_в \omega C + \operatorname{tg} kL}$$

б) $z_н = \dfrac{i\omega L}{c^2}$, $|\Gamma| = \dfrac{|i\omega L - c^2 z_в|}{|i\omega L + c^2 z_в|} = 1$, $Z(-L) = i z_в \dfrac{\omega L + c^2 Z_в \operatorname{tg} kL}{c^2 Z_в - \omega L \operatorname{tg} kL}$

в) $z_н = R = z_в$, $\Gamma = 0$, $Z(-L) = z_в$

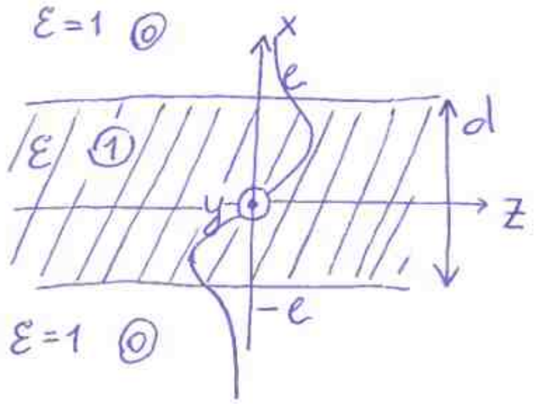
г) $z_н = z_{в_1}$, $\Gamma = \dfrac{z_{в_1} - z_в}{z_{в_1} + z_в}$, $Z(-L) = Z_в \dfrac{z_{в_1} + i z_в \operatorname{tg} kL}{z_в + i z_{в_1} \operatorname{tg} kL}$

д) $z_н = 0$, $\Gamma = -1$, $Z(-L) = i z_в \operatorname{tg} kL$

е) $z_н = \infty$, $\Gamma = 1$, $Z(-L) = -i Z_в \operatorname{ctg} kL$

№ 10.29

$\mu = 1$.



$$\frac{d^2\psi^m}{dx^2} + \varkappa_0^2 \psi^m = 0$$

$h^2 = k_0^2 - \varkappa_0^2$, $\varkappa_0^2 = -p^2$.

$|x| > l$: $\begin{cases} \psi^m = B_1 e^{-px}, & x > l \\ \psi^m = B_2 e^{px}, & x < -l \end{cases}$

$$\frac{d^2\psi^m}{dx^2} + \varkappa_1^2 \psi^m = 0.$$

$h^2 = k_1^2 - \varkappa_1^2$, $k_1 = k_0\sqrt{\varepsilon}$

$|x| < l$: $\psi^m = A_1 \cos \varkappa_1 x + A_2 \sin \varkappa_1 x$.

$k_0^2 - \varkappa_0^2 = k_1^2 - \varkappa_1^2$, $\underline{\varkappa_1^2 + p^2 = k_0^2(\varepsilon - 1)}$

$E_y(x) = -E_y(-x)$ по условию $\Rightarrow \psi^m(x) = -\psi^m(-x)$

$\Rightarrow \psi^m(x) = A \sin \varkappa_1 x$

TE:

$$\left.\begin{aligned} H_z &= \frac{\varkappa^2}{ik_0\varepsilon\mu}\psi^m \\ \bar{H}_\perp &= -\frac{h}{k_0\varepsilon\mu}\nabla_\perp\psi^m \\ \bar{E}_\perp &= -\frac{1}{\varepsilon}[\nabla_\perp\psi^m, \bar{z}^0] \end{aligned}\right\} e^{i(\omega t - hz)}$$

Удовлетворим г.у. $E_\tau = E_y$, $H_\tau = H_z$ при $x = l$:

$$\begin{cases} \dfrac{\varkappa_1^2 A \sin\varkappa_1 l}{\varepsilon} = -p^2 B e^{-pl}, \\ \dfrac{\varkappa_1 A \cos\varkappa_1 l}{\varepsilon} = -pBe^{-pl}. \end{cases} \Rightarrow \underline{\varkappa_1 \operatorname{tg} \varkappa_1 l = p}$$

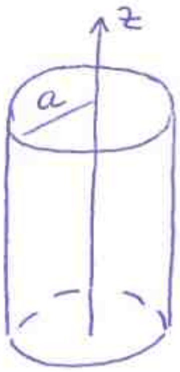
Если $k_0\sqrt{\varepsilon} \ll 1 \Rightarrow \varkappa_1 l \ll 1 \Rightarrow (\varkappa_1 l)^2 = pl$

$\varkappa_1^2 + p^2 = k_0^2(\varepsilon - 1)$, $pl + \underbrace{(p^2 l^2)}_{\to 0} = k_0^2 l^2 (\varepsilon - 1)$,

$p = k_0^2 l(\varepsilon - 1)$, $h^2 = k_0^2 + p^2 = k_0^2 + k_0^4 l^2 (\varepsilon - 1)^2 =$

$= k_0^2\left(1 + k_0^2 \frac{d^2}{4}(\varepsilon - 1)^2\right)$

№ 10.31.



$$\Delta_\perp \psi + \varkappa^2 \psi = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial \vartheta^2} + \varkappa^2\psi = 0.$$

$$\psi = R(r)\,\Theta(\vartheta)$$

$$r^2\frac{R''}{R} + r\frac{R'}{R} + \varkappa^2 r^2 + \frac{\Theta''}{\Theta} = 0$$

$$\begin{cases} r^2\dfrac{R''}{R} + r\dfrac{R'}{R} + \varkappa^2 r^2 = C_1, \\ \dfrac{\Theta''}{\Theta} = -C_1 . \end{cases}$$

$$\Theta = A_1\cos\sqrt{C_1}\,\vartheta + A_2\sin\sqrt{C_1}\,\vartheta, \quad \sqrt{C_1} = m, \; m = \overline{0,1\ldots}$$

$$R'' + \frac{R'}{r} + \left(\varkappa^2 - \frac{m^2}{r^2}\right)R = 0$$

$$R = B_1 J_m(\varkappa r) + B_2 N_m(\varkappa r), \quad B_2 = 0.$$

$$R(r) = B_1 J_m(\varkappa r)$$

$$\psi_m = J_m(\varkappa r)\begin{pmatrix}\cos m\vartheta \\ \sin m\vartheta\end{pmatrix}$$

TE: $\left.\dfrac{\partial \psi^m}{\partial r}\right|_a = 0 \Rightarrow J_m'(\varkappa a) = 0, \; \varkappa a = \mu_{mn}, \; \varkappa_{mn} = \dfrac{\mu_{mn}}{a}$

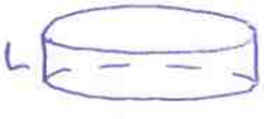
TM: $\left.\psi^e\right|_a = 0 \Rightarrow J_m(\varkappa a) = 0, \; \varkappa a = \nu_{mn}, \; \varkappa_{mn} = \dfrac{\nu_{mn}}{a}$

а) $L \gg a$.

$\varepsilon, \mu = 1, \; k^2 = \dfrac{\omega^2}{c^2} = h_p^2 + \varkappa_{mn}^2, \; \omega_{mnp}^2 = c^2\left(\varkappa_{mn}^2 + \left(\dfrac{p\pi}{L}\right)^2\right)$

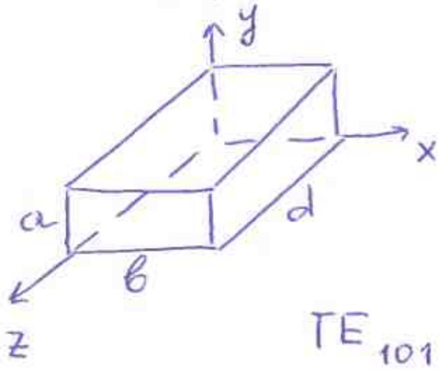
Самая низшая мода TE_{111}: $\omega_{111} = c\sqrt{\left(\dfrac{\mu_{11}}{a}\right)^2 + \left(\dfrac{\pi}{L}\right)^2}$

б) $L \ll a$



Самая низшая мода TM_{010}: $\omega_{010} = c\,\dfrac{\nu_{01}}{a}$

№ 10.33.



$\varepsilon, \mu = 1$, $k^2 = (\varkappa_{mn}^2 + (\frac{p\pi}{d})^2)$,

$\varkappa_{mn}^2 = \sqrt{(\frac{m\pi}{b})^2 + (\frac{n\pi}{a})^2}$, $k^2 = \frac{\omega^2}{c^2}$,

$\omega_{mnp}^2 = c^2((\frac{m\pi}{b})^2 + (\frac{n\pi}{a})^2 + (\frac{p\pi}{d})^2)$

TE_{101}: $\omega_{101} = \sqrt{(\frac{\pi}{b})^2 + (\frac{\pi}{d})^2}$

TM_{110}: $\omega_{110} = \sqrt{(\frac{\pi}{b})^2 + (\frac{\pi}{a})^2}$

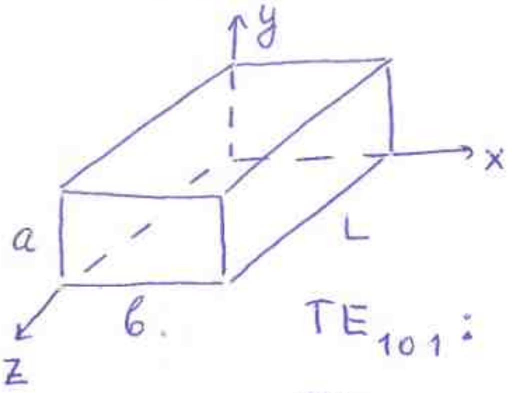
$\omega_{101} < \omega_{110} \Rightarrow$ низшей модой будет TE_{101}

$\bar{E}_{\perp} = \bar{y}_0 E_0 \sin\frac{\pi x}{b} \sin\frac{\pi z}{d} e^{i\omega t}$

$W = \bar{W}^e + \bar{W}^m = \iiint \frac{\varepsilon |\bar{E}|^2}{16\pi} dV + \iiint \frac{\mu |\bar{H}|^2}{16\pi} dV =$

$= \frac{1}{8\pi} \int_0^a \int_0^b \int_0^d |\bar{E}_{\perp}|^2 dx dy dz = \frac{abd}{32\pi} E_0^2$

N 10.35 (a)



$\varepsilon, \mu = 1$, $K^2 = \left(\varkappa_{mn}^2 + \left(\frac{p\pi}{L}\right)^2\right)$,

$\varkappa_{mn}^2 = \sqrt{\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2}$, $K^2 = \frac{\omega^2}{c^2}$,

$$\omega_{mnp}^2 = c^2\left(\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{L}\right)^2\right)$$

TE_{101}: $\omega_{101} = \sqrt{\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{L}\right)^2}$

TM_{110}: $\omega_{110} = \sqrt{\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{a}\right)^2}$

$\omega_{101} < \omega_{110} \Rightarrow$ низшей модой будет TE_{101}

$$\bar{E}_\perp = \bar{y}_0 E_0 \sin\frac{\pi x}{b} \sin\frac{\pi z}{L} e^{i\omega_{101} t},$$

$$\operatorname{rot}\bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t}, \quad \bar{H} = \frac{ic}{\omega_{101}}\operatorname{rot}\bar{E}.$$

$$\operatorname{rot}\bar{E} = \begin{vmatrix} \bar{x}^0 & \bar{y}^0 & \bar{z}^0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_\perp & 0 \end{vmatrix} = -\bar{x}^0 \frac{\pi}{L} E_0 \sin\frac{\pi x}{b}\cos\frac{\pi z}{L} e^{i\omega_{101} t} +$$

$$+ \bar{z}^0 \frac{\pi}{b} E_0 \cos\frac{\pi x}{b}\sin\frac{\pi z}{L} e^{i\omega_{101} t}$$

$$\bar{H}_\perp = -\bar{x}^0 \frac{\pi}{L} E_0 \sin\frac{\pi x}{b}\cos\frac{\pi z}{L} \cdot \frac{ic}{\omega_{101}} e^{i\omega_{101} t}$$

$$W = \bar{W}^e + \bar{W}^m = \iiint \frac{\varepsilon|\bar{E}|^2}{16\pi} dV + \iiint \frac{\mu|\bar{H}|^2}{16\pi} dV =$$

$$= \frac{1}{8\pi}\int_0^a\int_0^b\int_0^L |\bar{E}_\perp|^2 dx\,dy\,dz = \frac{abL}{32\pi} E_0^2$$

$P_{ст} = \frac{c}{8\pi} \operatorname{Re}\zeta_{пов} \oint_S |\bar{H}_\tau|^2 ds$, в нашем случае $\bar{H}_\tau = \bar{H}_\perp$

$$P_{ст} = \frac{c}{8\pi}\sqrt{\frac{\omega_{101}}{8\pi\sigma}} \frac{c^2}{\omega_{101}^2}\frac{\pi^2}{L^2} E_0^2 \underbrace{\cos^2\frac{\pi z}{L}}_{= 1 \text{ при } z=L} a\int_0^b \sin^2\frac{\pi x}{b} dx =$$

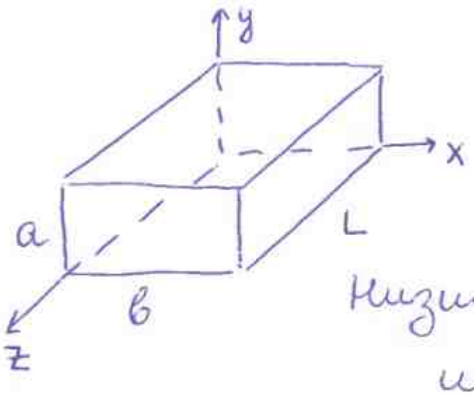
$$= \frac{c^3}{16\pi}\sqrt{\frac{\omega_{101}}{8\pi\sigma}}\frac{\pi^2}{L^2}\frac{E_0^2 ab}{\omega_{101}^2}, \quad w'' = \frac{P_{ст}}{2W} = \frac{c^3}{\omega_{101}^2}\frac{\pi^2}{L^3}\sqrt{\frac{\omega_{101}}{8\pi\sigma}} =$$

$$= \frac{c^3\pi^2}{c^2\left(\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{L}\right)^2\right)L^3}\sqrt{\frac{\omega_{101}}{8\pi\sigma}} = \sqrt{\frac{\omega_{101}}{8\pi\sigma}}\frac{cb^2}{(b^2+L^2)L}$$

$$Q = \frac{\omega_{101}}{2w''}$$

N 10.36.

$a < b < L$



$\mu = 1$

$\varepsilon = \varepsilon_r - i\varepsilon_i$

$\omega = \frac{kc}{\sqrt{\varepsilon\mu}}$, $\varepsilon = \varepsilon' + i\varepsilon'' \Rightarrow \omega = \omega' + i\omega''$

$$k^2 = \frac{\omega^2}{c^2}\varepsilon\mu = \left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2 + \left(\frac{p\pi}{L}\right)^2$$

Низшей модой будет TE_{101}:

$$\omega^2\varepsilon = c^2\left(\left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{L}\right)^2\right) = c^2k^2$$

$$\omega^2 = \omega'^2 - \omega''^2 + 2i\omega'\omega''$$

$$\omega^2 = \frac{c^2k^2}{\varepsilon_r - i\varepsilon_i} = \frac{c^2(\varepsilon_r + i\varepsilon_i)k^2}{\varepsilon_r^2 + \varepsilon_i^2}$$

$$\begin{cases} 2\omega'\omega'' = \dfrac{c^2\varepsilon_i k^2}{\varepsilon_r^2 + \varepsilon_i^2} \approx \dfrac{c^2\varepsilon_i k^2}{\varepsilon_r^2}, \\ \omega'^2 - \omega''^2 = \dfrac{c^2\varepsilon_r k^2}{\varepsilon_r^2 + \varepsilon_i^2} \approx \dfrac{c^2\varepsilon_r k^2}{\varepsilon_r^2} = \dfrac{c^2k^2}{\varepsilon_r}. \end{cases}$$

$\omega''^2 \ll \omega'^2$

$$\begin{cases} \omega'' = \dfrac{c^2\varepsilon_i k^2}{2\omega'\varepsilon_r^2}, \\ \omega'^2 = \dfrac{c^2k^2}{\varepsilon_r}. \end{cases} \Rightarrow \omega'' = \frac{\varepsilon_i\omega'^2}{2\omega'\varepsilon_r} = \frac{\varepsilon_i\omega'}{2\varepsilon_r}$$

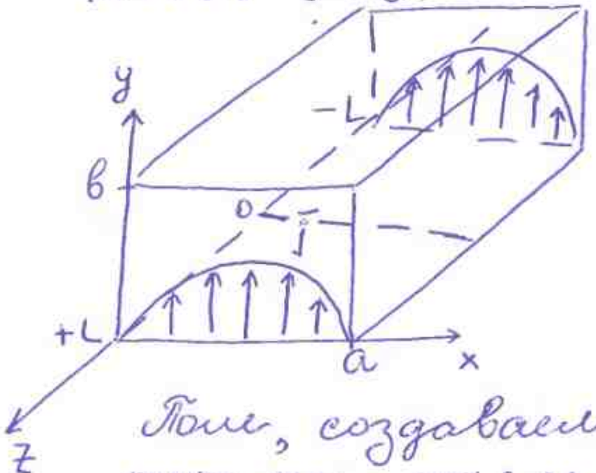
$$Q = \frac{\omega'}{2\omega''} = \frac{\varepsilon_r}{\varepsilon_i}$$

стр 17

№ 10.38

$|z|<L: \quad \bar{j} = \bar{y}_0 j_0 \sin\left(\frac{\pi x}{a}\right) e^{i(\omega t - hz)}, \quad h^2 = \frac{\omega^2}{c^2} - \left(\frac{\pi}{a}\right)^2$

$|z|>L: \quad \bar{j} = 0$



Данное распределение плотности тока возбуждает в волноводе волну TE_{10}:

$$\bar{E}_\perp = \bar{y}_0 E_0 \sin\frac{\pi x}{a} e^{i(\omega t - hz)},$$

$$h^2 = h_{10}^2 = \frac{\omega^2}{c^2} - \left(\frac{\pi}{a}\right)^2$$

Поле, создаваемое заданными сторонними токами, ищем в виде суперпозиции собственных колебаний волновода:

$$z>L: \quad \bar{E} = \sum_{p=1}^{\infty} a_p \bar{E}_p = a_{10}\bar{E}_{10}, \quad \bar{H} = a_{10}\bar{H}_{10}$$

$$z<-L: \quad \bar{E} = \sum_{p=1}^{\infty} a_{-p}\bar{E}_{-p} = a_{-10}\bar{E}_{-10}, \quad \bar{H} = a_{-10}\bar{H}_{-10}$$

$$\bar{E}_{\pm 10} = \bar{y}_0 E_0 \sin\frac{\pi x}{a} e^{\mp i h_{10} z}, \quad \bar{H}_{\pm 10} = \frac{ic \operatorname{rot} \bar{E}_{\pm 10}}{\omega\mu}.$$

$$P_+ = \frac{c}{8\pi} \operatorname{Re}\left(\frac{1}{\zeta_{\perp в}}\right) \iint |\bar{E}_{\perp +}|^2 ds = |a_{+10}|^2 \frac{c}{8\pi} \operatorname{Re}\left(\frac{1}{\zeta_{\perp в}}\right) \iint |\bar{E}_{+10}|^2 ds$$

$$P_- = |a_{-10}|^2 \frac{c}{8\pi} \operatorname{Re}\left(\frac{1}{\zeta_{\perp в}}\right) \iint |\bar{E}_{-10}|^2 ds$$

$$|\bar{E}_{+10}|^2 = |\bar{E}_{-10}|^2, \quad \frac{P_+}{P_-} = \frac{|a_{+10}|^2}{|a_{-10}|^2}$$

Определим коэф. возбуждения:

$$a_{+10} = \frac{1}{N_{10}} \iiint \bar{j}\,\bar{E}_{-10}\, dV = \frac{1}{N_{10}} \int_0^a \int_0^в \int_{-L}^{L} E_0 j_0 \sin^2\frac{\pi x}{a}\, dx\,dy\,dz =$$

$$= \frac{j_0 E_0}{N_{10}} \frac{a}{2} в 2L = \frac{a в L j_0}{N_{10}} E_0$$

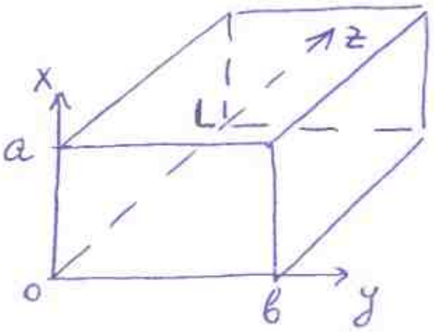
$$a_{-10} = \frac{1}{N_{10}} \iiint \bar{j}\,\bar{E}_{+10}\, dV = \frac{j_0 E_0}{N_{10}} \int_0^a \int_0^в \int_{-L}^{L} \sin^2\frac{\pi x}{a} e^{-2ih_{10}z}\, dx\,dy\,dz =$$

$$= \frac{j_0 E_0}{N_{10}} \frac{a}{2} в \left. \frac{e^{-2ih_{10}z}}{-2ih_{10}} \right|_{-L}^{L} = \frac{j_0 E_0 a в L}{N_{10}} \cdot \frac{\sin 2hL}{2hL}$$

$$\frac{P_+}{P_-} = \left(\frac{2hL}{\sin 2hL}\right)^2;$$ если $hL \ll 1$ ($L \ll \lambda$) $\Rightarrow \frac{P_+}{P_-} = 1$; если $hL \gg 1$, $\frac{P_+}{P_-} \gg 1$

N 10.48 (a) стр 18

$\bar{j} = \bar{x}_0\, j(y,z)\, e^{i\omega t}$



а) $\bar{j} = \bar{x}_0 j_0 \sin\frac{\pi y}{b} \sin\frac{\pi z}{L} e^{i\omega t}$
такое распределение плотности тока возбуждает низшую моду TE_{101}:

$\bar{E}_\perp^{101} = \bar{x}_0 E_0 \sin\frac{\pi y}{b} \sin\frac{\pi z}{L} e^{i\omega t}$

$\bar{E} = \bar{E}_{вих} + \bar{E}_{пот}$; $\bar{E}_{вих} = \sum\limits_{p=1}^{\infty} e_p \bar{E}_p = e_{101}\bar{E}_{101}$

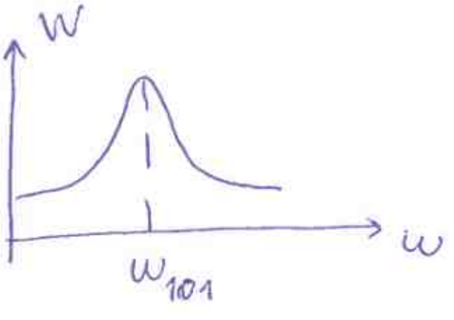
$$e_{101} = \frac{1}{(\omega^2-\omega_{101}^2)}\frac{1}{N_{101}} \iiint \omega \bar{j}\,\bar{E}_{101}\, dV =$$

$$= \frac{1}{(\omega^2-\omega_{101}^2)}\frac{1}{N_{101}} \int_0^a\int_0^b\int_0^L \omega j_0 E_0 \sin^2\frac{\pi y}{b}\sin^2\frac{\pi z}{L}\, dx\,dy\,dz =$$

$$= \frac{\omega j_0 E_0\, abL}{(\omega^2-\omega_{101}^2)\, 4 N_{101}}$$

$$W = \frac{\varepsilon}{8\pi} e_{101}^2 \iiint |\bar{E}_{101}|^2 dV = \frac{\varepsilon E_0^2}{8\pi} e_{101}^2 \int_0^a\int_0^b\int_0^L \sin^2\frac{\pi y}{b}\sin^2\frac{\pi z}{L}\, dx\,dy\,dz$$

$$= \frac{\varepsilon\, e_{101}^2}{32\pi} abL E_0^2$$



N 10.48 (б)

б) $$\bar{j} = \underbrace{\bar{x}_0\, j_0 \sin\frac{\pi y}{b}\sin\frac{\pi z}{L}e^{i\omega t}}_{\text{возбуждает моду } TE_{101}} + \underbrace{\bar{x}_0\, j_0 \sin\frac{\pi y}{b}\sin\frac{2\pi z}{L}e^{i\omega t}}_{\text{возбуждает моду } TE_{102}}$$

$$\bar{E}_{вых} = \sum_{p=1}^{\infty} e_p \bar{E}_p = e_{101}\bar{E}_{101} + e_{102}\bar{E}_{102}.$$

$$\bar{E}_{101} = \bar{x}_0 E_0 \sin\frac{\pi y}{b}\sin\frac{\pi z}{L}e^{i\omega t}$$
$$\bar{E}_{102} = \bar{x}_0 E_0 \sin\frac{\pi y}{b}\sin\frac{2\pi z}{L}e^{i\omega t}$$

$$e_{101} = \frac{1}{(\omega^2-\omega_{101}^2)}\frac{1}{N_{101}}\iiint \omega\, \bar{j}\, \bar{E}_{101}\, dV =$$

$$= \frac{1}{(\omega^2-\omega_{101}^2)}\frac{j_0 E_0 \omega}{N_{101}}\int_0^a\int_0^b\int_0^L\left(\sin^2\frac{\pi y}{b}\sin^2\frac{\pi z}{L} + \sin^2\frac{\pi y}{b}\sin\frac{2\pi z}{L}\cdot\right.$$
$$\left.\cdot\sin\frac{\pi z}{L}\right)dx\,dy\,dz = \frac{\omega j_0 E_0 abL}{(\omega^2-\omega_{101}^2)\,4N_{101}}$$

$$\int_0^L \sin\frac{2\pi z}{L}\sin\frac{\pi z}{L}\,dz = 0\ !$$

$$e_{102} = \frac{1}{(\omega^2-\omega_{102}^2)}\frac{1}{N_{102}}\iiint \omega\, \bar{j}\, \bar{E}_{102}\, dV =$$

$$= \frac{1}{(\omega^2-\omega_{102}^2)}\frac{j_0 E_0 \omega}{N_{102}}\int_0^a\int_0^b\int_0^L\left(\sin^2\frac{\pi y}{b}\sin\frac{\pi z}{L}\sin\frac{2\pi z}{L} + \right.$$
$$\left.+\sin^2\frac{\pi y}{b}\sin^2\frac{2\pi z}{L}\right)dx\,dy\,dz = \frac{\omega j_0 E_0 abL}{(\omega^2-\omega_{102}^2)\,4N_{102}}.$$

$$W = \frac{\varepsilon}{8\pi}\iiint\left(|\bar{E}_{101}|^2 e_{101}^2 + |\bar{E}_{102}|^2 e_{102}^2\right)dV = \frac{\varepsilon abL E_0^2}{32\pi}\left(e_{101}^2 + e_{102}^2\right)$$